CAUSALITY AND TIME: AN ULTRAMETRIC VIEW

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Abstract. In the talk, we develop an ultrametric (to be more specific, a p-adic) approach to causality and time. The goal is to finally construct a mathematical theory of functions which agrees with the current physical models of causality and time both at macro- and Planck scales.

Common physical models are mostly based on Archimedean time and space which in turn are based on the assumption that any temporal or spatial interval can be divided into smaller intervals 'ad infinitum'. However, if one assumes that Planck time and Planck length are the smallest intervals which can not be divided into smaller ones, then it is reasonable to try to construct a mathematical theory which starts with that 'indivisible' values accompanied by the assumption that total amount of that values can be increased 'ad infinitum'. In both cases, the 'infinity' just stands for a value which is extremely small (or extremely large) compared to the given value so that calculations involving the notion of infinity result in values which agree with respective measured values up to a small real number, the error. Therefore if the both theories (the one which assumes division of an interval into smaller intervals 'ad infinitum', and another one which assumes summing of 'indivisible' values 'ad infinitum') adequately reflect physical reality, the both theories must 'meet one another somewhere in the middle of the scale'. Up to normalization, one may assume that Plank units have value 1, then, e.g., 'movement' can be considered as a sequence of 'elementary steps' each of 1 spatial unit long, i.e., as a sequence of 1's and 0's where i-th term is 1 (respectively, 0) stands for the case a particle moves/does not move during 1 temporal unit. If one assumes that a physical system is causal, then the physical system is just an automaton which accepts the value at every 'elementary step', changes its 'internal state' to a newer one according to the value of the 'elementary step' and current state, and produces/does not produce the 'elementary effect', i.e., say, 1 or 0. The infinite strings can naturally be associated to 2-adic integers and the automata to mappings from $\mathbb{Z}_2$ to $\mathbb{Z}_2$ which satisfy a Lipschitz condition with a constant 1 w.r.t. 2-adic metric (1-Lipschitz, for brevity); and vice versa, every 1-Lipschitz mapping $\mathbb{Z}_2 \to \mathbb{Z}_2$ corresponds to a causal physical system, an automaton. Being 1-Lipschitz (thus, continuous) w.r.t. 2-adic metric, the automata mappings can be completely determined by the values which they take at any dense subset of $\mathbb{Z}_2$. The subset $\mathbb{Z}_2 \cap \mathbb{Q}$ of rational 2-adic integers is dense both in $\mathbb{Z}_2$ w.r.t 2-adic metric and in $\mathbb{R}$ w.r.t. standard metric of the field of real numbers $\mathbb{R}$. Thus one can ask what automata functions $f\colon \mathbb{Z}_2 \to \mathbb{Z}_2$ can be uniquely expanded to functions $\mathbb{R} \to \mathbb{R}$ which are continuous real-valued functions of real variable. That class $\mathcal{C}_2(\mathbb{R})$ of real functions which are so defined by the class $\mathcal{C}(\mathbb{Z}_2)$ of causal 2-adic functions turns out to be rather wide; for instance, it contains all polynomials with rational 2-adic integer coefficients (i.e., the polynomials whose coefficients are in $\mathbb{Z}_2 \cap \mathbb{Q}$), all rational functions $u(x)/1 + 2w(x)^2$ where $u(x), v(x) \in \mathbb{Z}[x]$ are arbitrary polynomials with integer coefficients, $\mathbb{Z} = \{0, \pm1, \pm2, \ldots\}$, etc. Functions from the class $\mathcal{C}_2(\mathbb{R})$ exhibit some properties which might be considered as 'physical' such as, e.g., a hologram-like property: Any function from $\mathcal{C}_2(\mathbb{R})$ is uniquely defined by its values at the (rational 2-adic integer) points from any (thus, any arbitrarily small) real interval $(\alpha, \beta) \subset \mathbb{R}$.

As any continuous function $\mathbb{R} \to \mathbb{R}$ can be uniformly approximated by functions from $\mathcal{C}_2(\mathbb{R})$ on any segment $[\alpha, \beta] \subset \mathbb{R}$, the class is $\mathcal{C}_2(\mathbb{R})$ a good candidate to represent causality and time both at macro- and Planck scales.

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